La matematica per l'innovazione digitale nei processi industriali

Luca Formaggia

MOX, Dipartimento di Matematica, Politecnico di Milano

Presidente della Società Italiana di Matematica Applicata ed Industriale

SIMAI SOCIETÀ ITALIANA DI MATEMATICA APPLICATA E INDUSTRIALE

MODELLISTICA E CALCOLO SCIENTIFICO MOX MODELING AND SCIENTIFIC COMPUTING

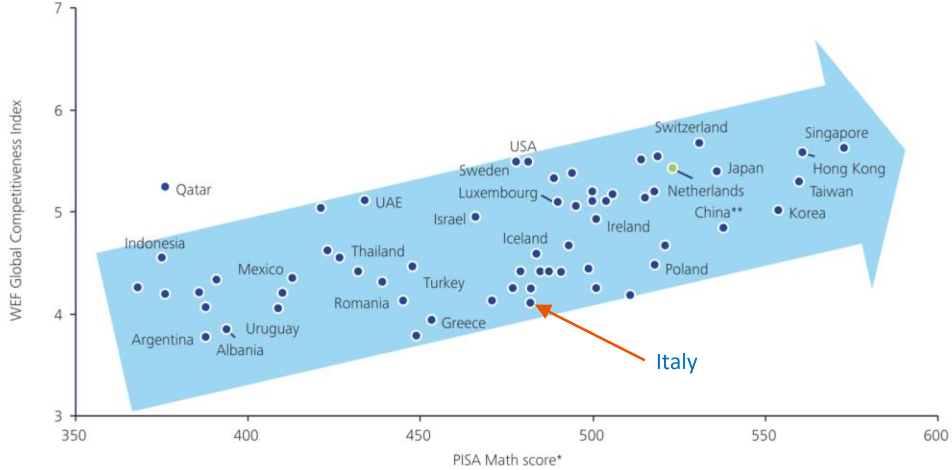
OVERVIEW

- The impact of mathematics in industry and economy
- Some challenges for the numerical modeller
- The next challenges

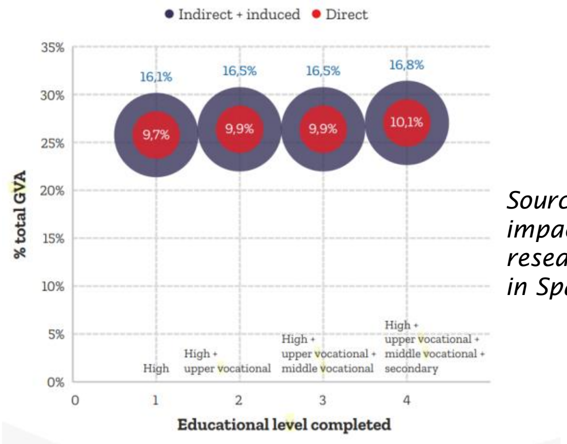
The impact of mathematics in industry and economy

	Employment (×1000)				Gross Value Added (€ / £)			
	Direct	Indirect	Induced	Total	Direct	Indirect	Induced	Total
NL	928	542	789	**2,259**	€ 71bn	€ 37bn	€ 51bn	**€ 159bn**
	10.7%	6.2%	9.1%	**26%**	13.2%	6.9%	9.5%	**30%**
UK	2,800	2,900	4,100	**9,800**	£ 192bn	£ 155bn	£ 208bn	**£ 555bn**
	9.8%	10.2%	14.4%	**34%**	16%	12%	15%	**43%**

Source: Mathematical sciences and their values for the Dutch economy, Deloitte 2014

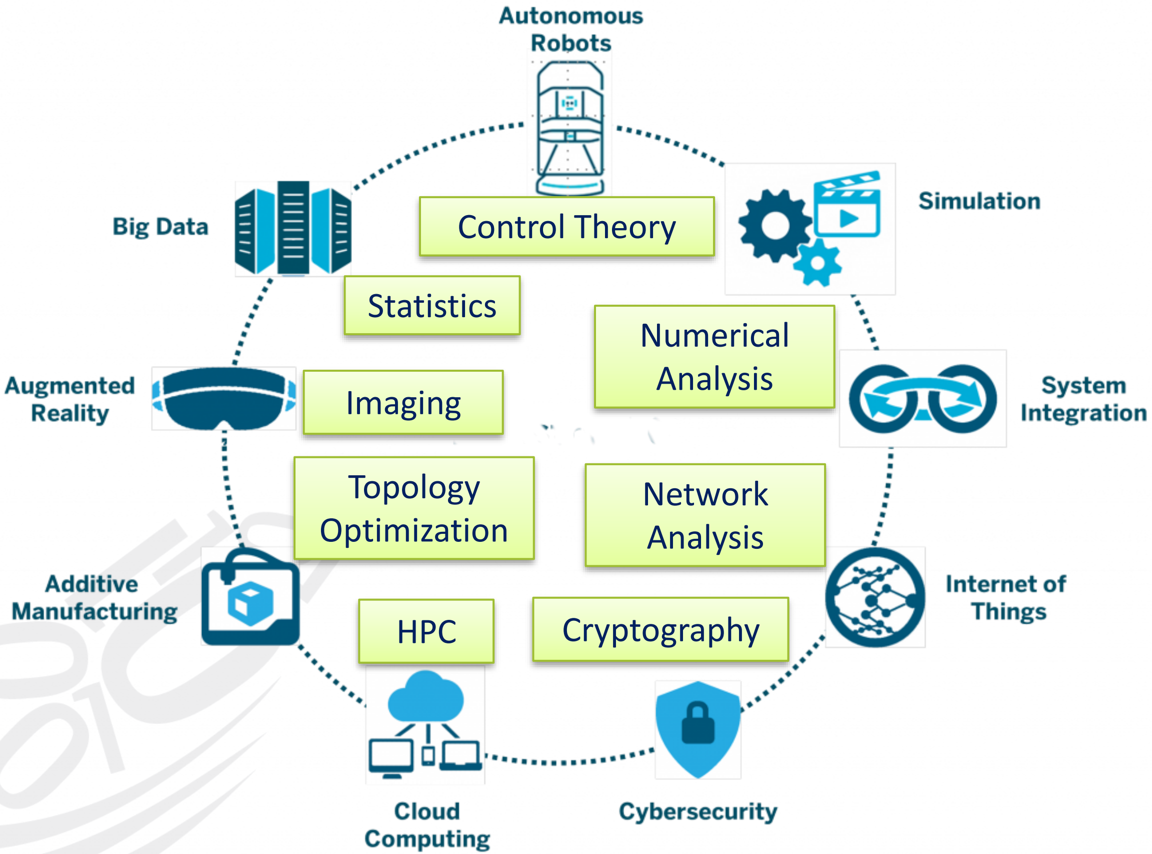


Competitiveness versus math score. Source: World Economic Forum and OECD-PISA, 2013



Source: Socioeconomic impact of mathematical research and technology in Spain, 2019

Mathematics in industry 4.0



Some challenges for the numerical modeller

Multi-physics phenomena

Optimization

Parameter estimation

Uncertainities and variability

Integration with the industrial workflow

Integration with data

Fast (but accurate) solvers

- Geometrical reduction
- Reduced Basis
- DL-ROM
- Mesh adaptivity
- Splitting schemes
- …

Math for energy transition
Reducing loss in submarine cables

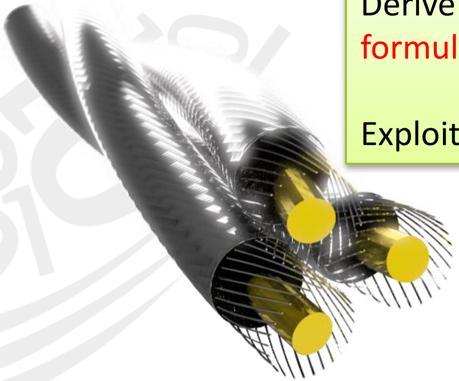
In collaboration with DEIB and Prysmian Group

The Industrial objectives: find the optimal cable configuration to reduce power lossess due to eddy currents and hysteresis

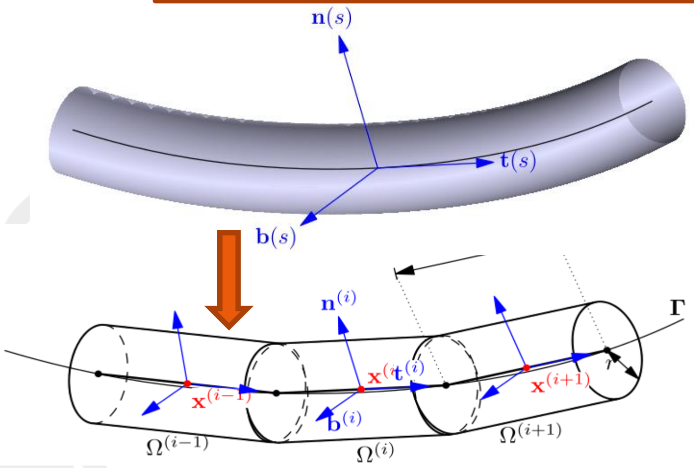
Standard 3D magnetostatic simulation too costly!

The answer:
Derive a 1D reduced model for the wire and develop an integral formulation for the problem

Exploit symmetries to develop a fast and parallel solver

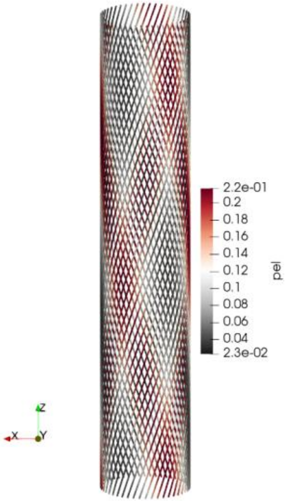


Reducing loss in submarine cables



$$\mathbf{M}_i = \frac{1}{\mu_0}(I - K^{-1})\left(\mathbf{B}_i^{ext} + \sum_j \mathbf{B}_{ij}^M\right)$$

$$\mathbf{B}^M(\mathbf{x}_i) = \sum_j \mathbf{B}_{ij}^M = \sum_j \frac{\mu_0}{4\pi}\int_{\partial\Omega_j}(\mathbf{n}\times\mathbf{M}_j)\times\frac{\mathbf{x}_i - \mathbf{x}}{|\mathbf{x}_i - \mathbf{x}|^2}$$

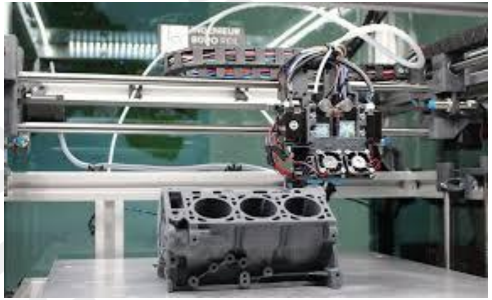


Losses due to eddy currents

Computational time		
	Contralay	Equilay
MoM	1.9 s	4.5 s
Standard Technique	8h	28h

4 orders of magnitude gain

Math for additive manufacturing
Mesh adaptation for topology optimization



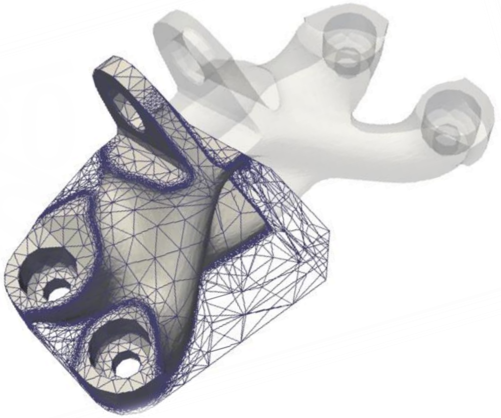
3D printing allows us to manufacture products with shapes which cannot be obtained with standard techniques.

This opens up new possibilities for conceiving innovative geometries via topology optimization

The **industrial objectives**: to find the best allocation of material inside to obtain the desired mechanical properties

The answer:
Develop procedures for topology optimization, leading the process to a free-form design with an optimal computational cost.

Mesh adaptation for topology optimization

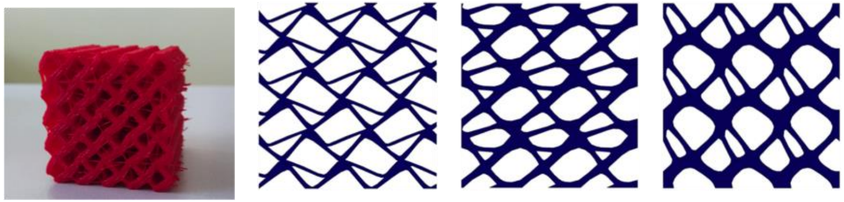


Topology optimization at the macro-scale

Find the optimal topology which ensures the target mechanical characteristics

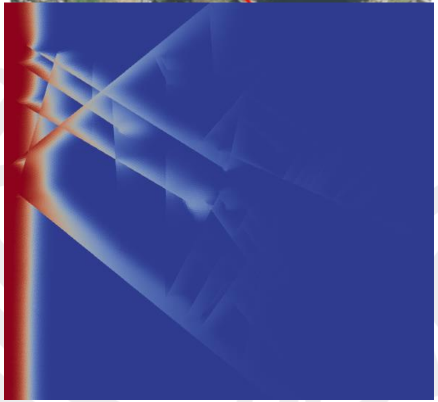
Topology optimization at the micro-scale

Find the unit cell that guarantees a desired physical property at the macroscale



Math for Energy
Optimize the exploitation of energy resources

In collaboration with eni



Fractures are ubiquitous in porous media and have an important effect on flow and transport.

Their presence must be accounted for to optimize hydrocarbon extraction, geothermal exploitation or assess CO2 storage facilities

The industrial objectives: introduce the explicit representation of fractures in the context of an existing complex industrial simulator and modeling workflow

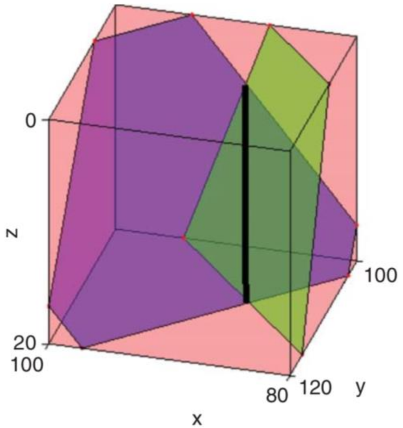
The answer

Develop embedded methods that use a regular background grid cut by fractures

Implement a minimally intrusive procedure to introduce the numerical technique in the existing workflow

Optimize the exploitation of energy resources

Geometrical preprocessor

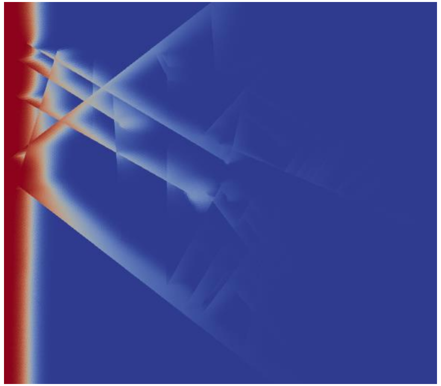


$$\langle d \rangle_{if} = \frac{1}{V_i} \int_{V_i} \left| \left(\mathbf{x} - \mathbf{x}_f \right) \cdot \mathbf{n}_f \right| dV$$

Black box solver
(nonlinear conservation laws)

$$\frac{dM^\kappa}{dt} + M^\kappa \nabla \cdot \mathbf{u_s} + \nabla \cdot \mathbf{F}^\kappa_{adv} = q$$

$$T_{if} = \frac{2A_{if}}{\langle d \rangle_{if}} \lambda_{if}$$



Flow field in a fractured media

Compute equivalent transmissibility by numerical upscaling

Math for industrial innovation

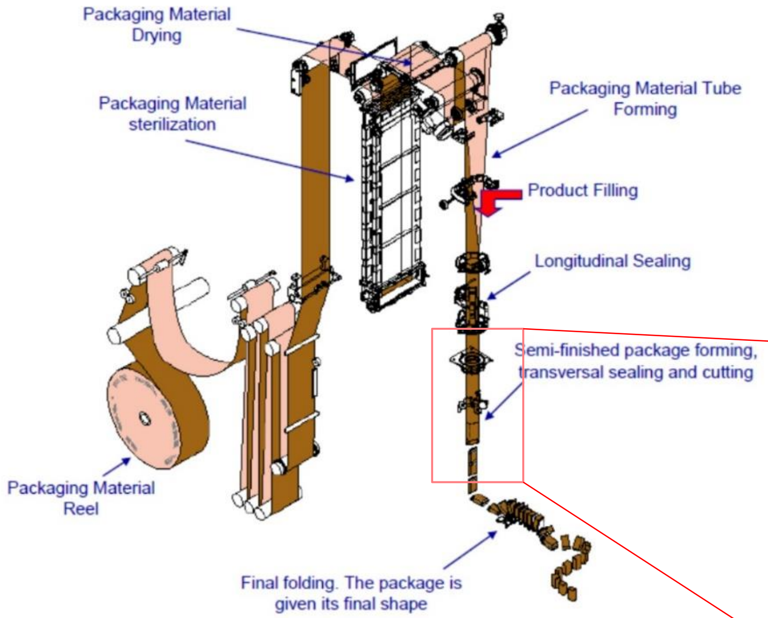
Model reduction for the packaging industry

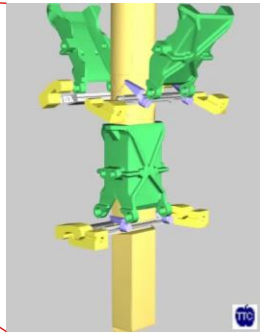
The industrial objective: control and optimize the packaging process for high production rate

In collaboration with

Tetra Pak — MOX OFF Mathematics for Innovation, Zucchetti Group

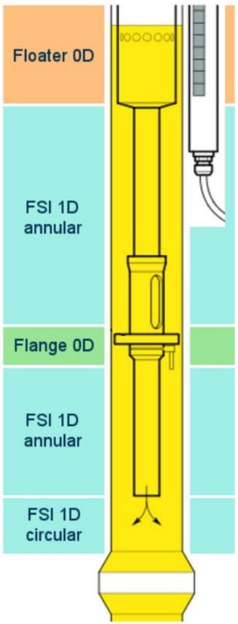
The answer: develop suitable reduced models to make the problem computationally tractable. Integrate the model in the control system.





Model reduction for the packaging industry

Hybrid dimensional model



FSI 1D

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial z} = 0$$

$$\frac{\partial Q}{\partial t} + \alpha \frac{\partial}{\partial z}\left(\frac{Q^2}{A}\right) + \frac{A}{\rho}\frac{\partial p}{\partial z} + K_r\left(\frac{Q}{A}\right) + Ag = 0$$

Circular section

$$\psi = \beta_0 \frac{\sqrt{A} - \sqrt{A_0}}{\sqrt{A_0}}$$

Ring section

$$\psi = \beta_0 \frac{\sqrt{A + c_i} - \sqrt{A_0 + c_i}}{\sqrt{A_0 + c_i}}$$

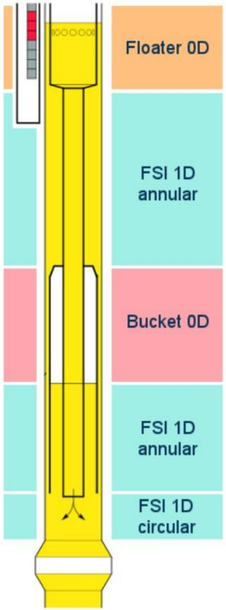
Flange 0D

$$Q_i(\Delta p) = C_{di} A_i \rho \, \mathrm{sign}(\Delta p) \sqrt{\frac{2}{\rho}|\Delta p|}$$

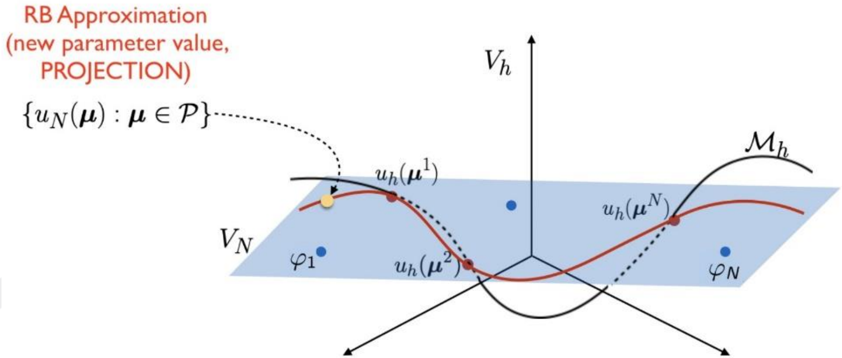
Bucket 0D

$$p_g = \rho_g R T$$

$$h_l(t) = \left(h_0 + \frac{1}{A}\int_0^t Q_{\Omega_0} \, d\tau\right)$$



Model reduction for the packaging industry
Reduced Order Model

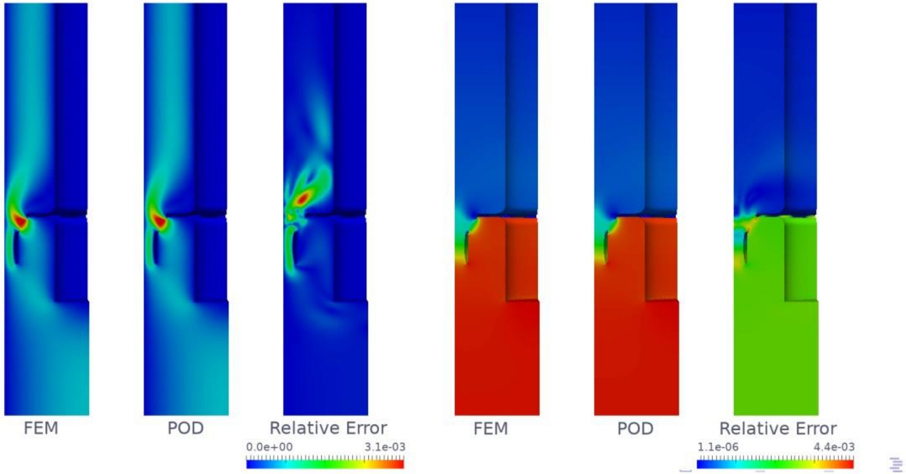


- Simulate different scenario by varying problem parameters at a strongly reduced computational costs
- Enable to implement multi-query procedure like numerical optimization or parameter estimation

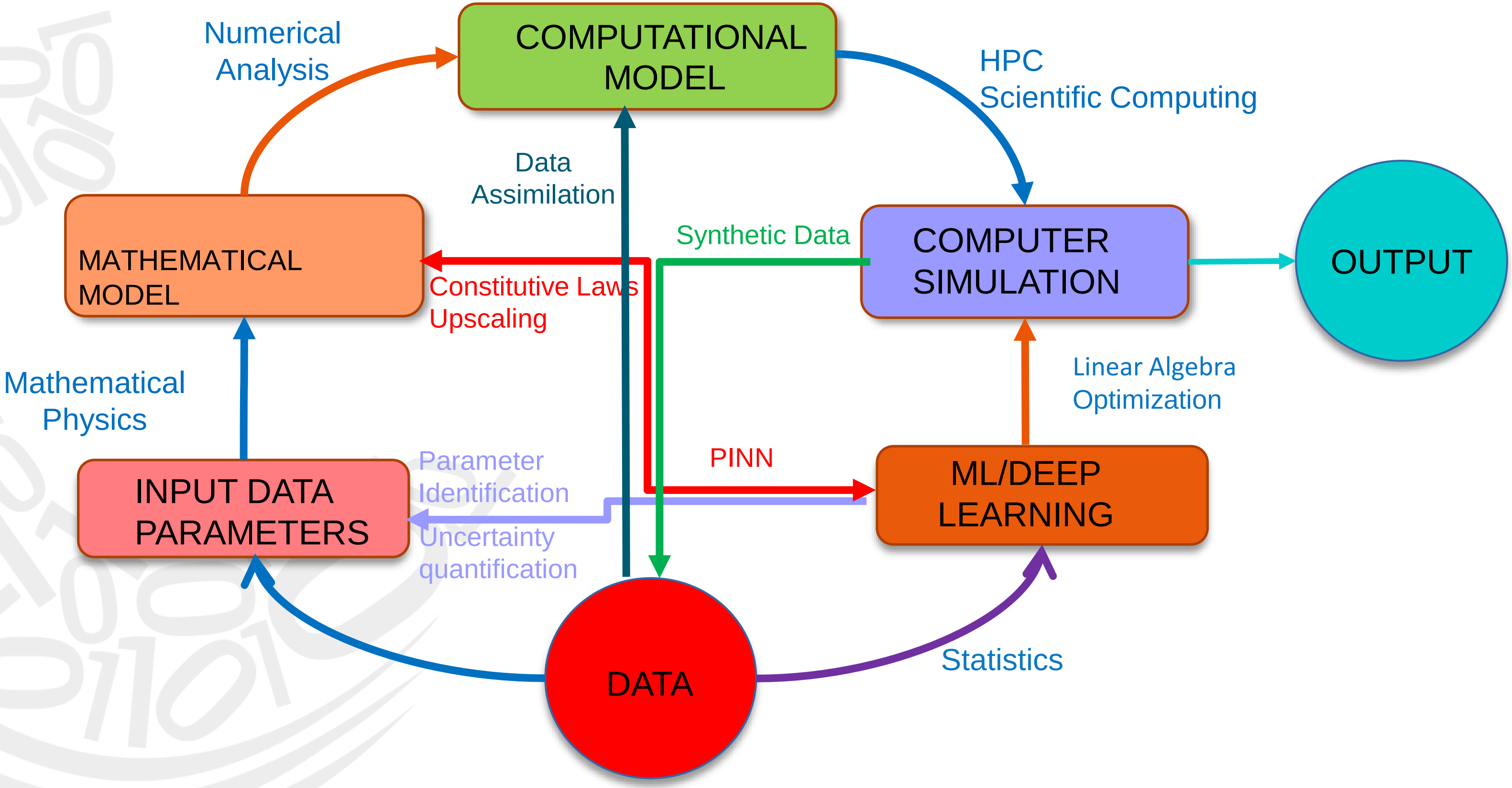
3D test comparison

- 3D incompressible stationary Navier-Stokes equations
- Parameters: viscosity and carton tube diameter
- $M = 24$ snapshots

Geometry	3D
Problem type	Stationary
# nodes	53K
# elements	263K
FOM CPU time	4.5 hours
RB CPU time (POD, N=5)	3.5 mins (75x)
RB CPU time (POD, N=10)	7.3 mins (37x)
RB CPU time (POD, N=15)	11.2 mins (24x)



A next challenge:
integrating data and physics-based simulations



Conclusioni

- L'uso di sofisticati strumenti matematici e tecniche computazionali può migliorare i processi industriali e la produzione, a condizione che possano essere integrati nel flusso di lavoro industriale e fornire risposte in tempi ragionevoli. Ciò richiede l'integrazione di competenze matematiche con la conoscenza del processo industriale.

- Una sfida per il futuro è l'integrazione tra modelli di simulazione basati sulla fisica, che enucleano le nostre conoscenze pregresse del fenomeno in esame, e modelli basati sui dati, in grado di superare la mancanza di conoscenza, o inglobare la variabilità, apprendendo dai dati.

- Ciò richiede competenze interdisciplinari che spaziano dalle tecniche numeriche, calcolo scientifico e statistica.

La SIMAI

- La Società Italiana di Matematica Applicata ed Industriale promuove la ricerca proponendo una visione della matematica applicata trasferibile dall'accademia all'industria.

- Favorisce il coordinamento dei programmi di studio e di ricerche promuovendo i contatti tra ambienti universitari, enti pubblici di ricerca e industrie.

- Organizza, seminari, congressi e workshop tematici e istituisce premi per ricercatori in matematica applicata o industriale.

- Fondata nel 1989, conta oggigiorno circa 350 membri.

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